

# phase plane analysis and statefinder diagnostic of agegraphic dark energy in 5D Brans-Dicke cosmology

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## Abstract

We present an autonomous phase-plane describing the evolution of field equations containing an agegraphic dark energy in 5D Brans- Dicke cosmology. To observationally verify the numerical results, we simultaneously solve the equations by constraining the model parameters with SNe Ia data. We find conditions for the existence and stability of the critical points (states of the universe) and numerically examine the cosmological parameters. We also investigate the model by means of statefinder diagnostic.

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## 1. INTRODUCTION

Various cosmological observations such as Cosmic Microwave Background (CMB) [1], [2], Supernova type Ia (SNIa)[3], [4], Weak Lensing[5], Baryon Acoustic Oscillations (BAO) [6], 2dF Galaxy Redshift Survey (2dFGRS) [7] at low redshift and DEEP2 redshift survey [8] at high redshift, have provided cross-checked data to determine cosmological parameters with high precision. These parameters imply that our approximately 13.7 year-old universe is nearly spatially flat, homogeneous and isotropic at large scale, i.e. a Friedmann-Robertson-Walker (FRW) universe with zero curvature, and has entered an accelerating phase since  $z \approx 0.46$  [4]. Moreover, according to  $\Lambda$ CDM model, the universe consists of 0.046 baryonic matter, 0.228 non-relativistic unknown matter, namely dark matter (DM), and a remarkable amount of 0.726 smoothly distributed dominant dark energy (DE) [2]. The equation of state (EoS) of DE, is the main parameter which determines the gravitational effect of DE on the evolution of the universe, and can be measured from observations without need to have a definite model of DE. Strong evidences imply that the EoS of DE lies in a narrow range around  $w \approx -1$  and has a smooth evolution [9], [10]. Theoretically, one can classify the EoS of DE with respect to the barrier  $w = -1$ , namely the phantom divide line (PDL) [11]. That is, DE with the EoS of  $w = -1$  employs the cosmological constant,  $\Lambda$ , with a constant energy density. The case with dynamical EoS where  $w \geq -1$ , is referred to as quintessence [12]. and  $w \leq -1$  corresponds to phantom energy [13], [14].

On the other hand, the problem of DE, its energy density and EoS parameter is still an unsolved problem in classical gravity and may be in the context of quantum gravity we achieve a more inclusive insight to its properties [15]. The holographic dark energy (HDE) model is an attempt to apply the nature of DE within the framework of quantum gravity [16], [17]. The holographic principle states that the number of degrees of freedom describing the physics inside a volume (including gravity) is bounded by the area of the boundary which encloses this volume and thus related to entropy scales with the enclosing area of the system [18]. Since the entropy scales like the area rather than the volume, the fundamental degrees of freedom describing the system are characterized by an effective quantum field theory in a box of size  $L$  with one fewer space dimensions and with planck-scale UV cut-off  $\Lambda$  [18].

Among all the cosmological models, the scalar-tensor theories have been widely used to explain the late time acceleration of the universe and its relation to the HDE [19]–[44].

One example is Brans-Dicke (BD) theory [45], where the gravitational constant becomes time dependent varying as inverse of a time dependent scalar field which couples to gravity with a coupling parameter  $\omega$ . Many of the cosmological problems can be successfully explained by using this theory. Alternatively, the higher dimensional theories may explain cosmic acceleration and phantom crossing [46], [47]. The existence of extra dimensions is required in various theories beyond the standard model of particle physics, especially in theories unifying gravity with the other fundamental forces, such as superstring and M theories [48]–[51].

While the successful HDE model explains the observational data and has been studied widely by the authors ( see for example [52]–[54]), more recently, a new dark energy model, dubbed agegraphic dark energy (ADE) model, has been proposed by Cai [55]. The ADE is also related to the holographic principle of quantum gravity and takes into account the uncertainty relation of quantum mechanics together with the gravitational effect in general relativity.

Here, in an attempt to integrate both the scalar tensor and higher dimensional theories, the 5-dim Brans-Dicke (BD) cosmology is studied with the aim to understand the agegraphic nature of dark energy in the model. We perform stability analysis and investigate the attractor solutions of the model by utilizing the 2-dimensional phase space of the theory. Also, we simultaneously best fit the model parameters with the observational data using  $\chi^2$  method. This enables us to find the best fitted model parameters for the analysis of the critical points and also verification of the model with the experiment. In addition, we use the well-known statefinder parameters [56] to differentiate among cosmological models. These parameters are used to explore a series of DE cosmological models [57]–[60]

## 2. THE MODEL

The 5-D Brans-Dicke action is given by,

$$S = \int d^5x \sqrt{{}^{(5)}g} \left( \frac{\phi^2}{8\omega} {}^{(5)}R - \frac{1}{2} g^{AB} \nabla_A \phi \nabla_B \phi + L_M \right), \quad (1)$$

where  $\omega$  is a dimensionless coupling constant which determines the coupling between gravity and BD scalar field,  $R$  is 5D Ricci scalar,  $\phi(x^A)$  is the BD scalar field and  $L_M$  is the

Lagrangian of matter field. We assume the metric in 5-D space-time in the form of,

$$ds^2 = -dt^2 + a^2(t, y) \left( \frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right) + b^2(t, y) dy^2, \quad (2)$$

where the 4-D space time is assumed to be homogeneous and isotropic (FRW universe). In the metric,  $k$  is the curvature parameter with  $k = -1, 0, 1$  corresponding to open, flat, and closed universes, respectively. The scalar field  $\phi$  and the scale factors  $a$  and  $b$ , in general are functions of  $t$  and  $y$ . For simplicity and plausibility, we assume the extra dimension is cyclic, i. e. the hypersurface-orthogonal space-like is a killing vector field in the underlying 5D space-time [61]. Hence, all fields are function of the cosmic time only. Note that, the functionality of scale factor  $b$  on  $y$ , either can be eliminated by transforming to a new extra coordinate if  $b$  is a separable function, and or makes no change in the following equations if  $b$  is the only field that depends on  $y$ . Beside, in the compacting extra dimension scenarios, all field are Fourier-expanding around  $y$ , and henceforth one can have terms independent of  $y$  to be observable, i. e. physics would thus be effectively independent of compactified fifth dimension [62].

Variation of the action (1) with respect to metric (2) yields the following field equation,

$$\frac{3}{4\omega} \phi^2 \left( H^2 + \frac{\dot{b}}{bH} H^2 + \frac{k}{a^2} \right) - \frac{1}{2} \dot{\phi}^2 + \frac{3}{2\omega} H \dot{\phi} \phi + \frac{1}{2\omega} \frac{\dot{b}}{bH} H \dot{\phi} \phi = \frac{\rho}{b} + \rho_B, \quad (3)$$

$$\begin{aligned} & \frac{-1}{4\omega} \phi^2 \left( 2\frac{\ddot{a}}{a} + H^2 + 2\frac{\dot{b}}{bH} H^2 + \frac{\ddot{b}}{b} + \frac{k}{a^2} \right) - \frac{1}{2\omega} (2H + \frac{\dot{b}}{bH} H) \dot{\phi} \phi - \frac{1}{2\omega} \ddot{\phi} \phi - \frac{1}{2} \left( \frac{1}{2} \right. \\ & \left. + \frac{1}{\omega} \right) \dot{\phi}^2 = p_B, \end{aligned} \quad (4)$$

$$-\frac{3}{4\omega} \phi^2 \left( 2H^2 + \dot{H} + \frac{k}{a^2} \right) - \frac{1}{4\omega} \phi \ddot{\phi} - \frac{1}{4} \dot{\phi}^2 \left( 1 + \frac{1}{\omega} \right) - \frac{3H}{4\omega} \phi \dot{\phi} = 0, \quad (5)$$

$$\ddot{\phi} + 3H \dot{\phi} + \frac{\dot{b}}{bH} H \dot{\phi} - \frac{3}{2\omega} \left( \frac{\ddot{a}}{a} + H^2 + \frac{k}{a^2} + \beta H^2 + \frac{\ddot{b}}{3b} \right) \phi = 0, \quad (6)$$

where  $H = \frac{\dot{a}}{a}$ . In the orthonormal basis  $e^0 = dt, e^i = adx^i$  and  $e^5 = bdy$ , the stress-energy tensor can be considered as.

$$T_B^A = T_B^A|_{bulk} + T_B^A|_{brane} \quad (7)$$

where  $T_B^A|_{bulk}$  is the energy momentum tensor of the bulk matter and

$$T_B^A|_{bulk} = \text{diag}(-\rho_B, p_B, p_B, p_B, q_B) \quad (8)$$

The second term corresponds to the matter content in the brane ( $y=0$ )

$$T_B^A|_{brane} = \text{diag}(-\rho, p, p, p, 0) \quad (9)$$

Assume that the 05 component of the energy-momentum tensor vanishes, which means that there is no flow of matter along the fifth dimension. Therefore the nonzero elements of the 5D stress-energy tensor are

$$T_{00} = \rho_B + \frac{\delta(y)}{b} \rho. \quad (10)$$

$$T_{ii} = a^2 \rho_B + a^2 \frac{\delta(y)}{b} p. \quad (11)$$

$$T_{55} = b^2 q_B. \quad (12)$$

We assume that the BD scalar field  $\phi$ , is in power law of the scale factor  $a(t)$ , in the form of  $\phi \propto a^\alpha$ . In the next section, we apply the ADE model in 5-D Brans-Dicke theory. The conservation equations for the dark energy and matter field in the universe are respectively,

$$\dot{\rho}_B + 3H(\rho_B + p_B) + \rho_B \left(\frac{\dot{b}}{b}\right) = 0, \quad (13)$$

$$\dot{\rho}_m + 3H\rho_m = 0, \quad (14)$$

The agegraphic dark energy model with the dark energy density is given by

$$\rho_\Lambda = \frac{3n^2 M_P^2}{T^2}, \quad (15)$$

where  $T$  is chosen to be the age of our universe and given by

$$T = \int dt = \int_0^a \frac{da}{Ha}. \quad (16)$$

In the framework of Brans-Dicke cosmology, we write the agegraphic energy density of the quantum fluctuations in the universe as

$$\rho_\Lambda = \frac{3n^2 \phi^2}{4\omega T^2}. \quad (17)$$

### 3. STABILITY ANALYSIS-PERTURBATION AND PHASE SPACE

The structure of the dynamical system can be studied via phase plane analysis, by introducing the following dimensionless variables,

$$\Omega_m = \frac{4\omega \rho_m}{3b\phi^2 H^2}, \quad \Omega_\Lambda = \frac{n^2}{H^2 T^2}, \quad \Omega_k = \frac{k}{H^2 a^2}, \quad \Omega_b = \frac{\dot{b}}{bH}, \quad (18)$$

The field equations (3)-(6) in terms of the new dynamical variables become,

$$\Omega'_m = -\Omega_m(3 + 2\alpha + \Omega_b + 2\frac{\dot{H}}{H^2}) \quad (19)$$

$$\Omega'_\Lambda = -2\Omega_\Lambda(\frac{\sqrt{\Omega_\Lambda}}{n} + \frac{\dot{H}}{H^2}) \quad (20)$$

$$\Omega'_k = -2\Omega_k(1 + \frac{\dot{H}}{H^2}) \quad (21)$$

$$\Omega'_b = \frac{\ddot{b}}{bH^2} - \Omega_b(\Omega_b + \frac{\dot{H}}{H^2}) \quad (22)$$

where prime " ' " means derivative with respect to  $\ln(a)$ . In the above equations we have

$$\frac{\dot{H}}{H^2} = -\frac{1}{1 + \frac{\alpha}{3}} \left( 1 + \frac{\alpha^2(2 + \omega)}{3} + \alpha + \Omega_k \right) \quad (23)$$

By using the Friedman constraint in terms of the new dynamical variables:

$$\Omega_k - \Omega_m - \Omega_\Lambda + 2\alpha(1 - \frac{\alpha\omega}{3}) + \Omega_b(1 + \frac{2\alpha}{3}) = 1 \quad (24)$$

the Eq. (19)-(22) reduce to,

$$\begin{aligned} \Omega'_m = & -\Omega_m \left[ 3 + 2\alpha + \Omega_b - \frac{2}{1 + \frac{\alpha}{3}} \left( 1 + \frac{\alpha^2(2 + \omega)}{3} + \alpha + \Omega_m + \Omega_\Lambda - 2\alpha(1 - \frac{\alpha\omega}{3}) \right. \right. \\ & \left. \left. - \Omega_b(1 + \frac{2\alpha}{3}) \right) \right] \end{aligned} \quad (25)$$

$$\begin{aligned} \Omega'_\Lambda = & -2\Omega_\Lambda \left[ \frac{\sqrt{\Omega_\Lambda}}{n} - \frac{1}{1 + \frac{\alpha}{3}} \left( 1 + \frac{\alpha^2(2 + \omega)}{3} + \alpha + \Omega_m + \Omega_\Lambda - 2\alpha(1 - \frac{\alpha\omega}{3}) \right. \right. \\ & \left. \left. - \Omega_b(1 + \frac{2\alpha}{3}) \right) \right] \end{aligned} \quad (26)$$

$$\begin{aligned} \Omega'_b = & 3(-1 - \frac{2\alpha}{3} + \frac{2\sqrt{\Omega_\Lambda}}{n} - \Omega_b)\Omega_\Lambda + 2 - \frac{2\alpha + 3}{1 + \frac{\alpha}{3}} \left( \right. \\ & - \frac{1}{1 + \frac{\alpha}{3}} \left( 1 + \frac{\alpha^2(2 + \omega)}{3} + \alpha + \Omega_m + \Omega_\Lambda - 2\alpha(1 - \frac{\alpha\omega}{3}) - \Omega_b(1 + \frac{2\alpha}{3}) \right) \left. \right) \\ & + 2\Omega_b(1 + \frac{2\alpha}{3}) + 2(2 + \Omega_b)\alpha + (4 + \omega)\alpha^2 - \Omega_b \left( \Omega_b \right. \\ & \left. - \frac{1}{1 + \frac{\alpha}{3}} \left( 1 + \frac{\alpha^2(2 + \omega)}{3} + \alpha + \Omega_m + \Omega_\Lambda - 2\alpha(1 - \frac{\alpha\omega}{3}) - \Omega_b(1 + \frac{2\alpha}{3}) \right) \right) \end{aligned} \quad (27)$$

It is more convenient to investigate the properties of the dynamical equations (25)-(27) than equations Eq. (19)-(22). In stability analysis, the above equations can be solved to find fixed points (critical points). These points are always exact constant solutions in the context of autonomous dynamical systems. The critical points in this model are highly nonlinear and depend on the stability parameters. In addition, in stability analysis the expressions for the

critical points and eigenvalues are long and cumbersome such that the usual procedure for stability analysis is not possible. To overcome the problem, we solve the above equations by best fitting the stability and model parameters and initial conditions with the observational data for distance modulus using the  $\chi^2$  method. This helps us to find the solutions for the above equations and conditions for the stability of the critical points that are physically more meaningful and observationally more favored. With the simultaneous stability analysis and best fitting, we find two critical points. The best fitted stability and model parameters and initial conditions are  $\alpha = 0.6$ ,  $\omega = -2.9$ ,  $n = 1.2$ ,  $\Omega_\Lambda(0) = .7$ ,  $\Omega_b(0) = .6$  and  $\Omega_m(0) = .27$ . The properties of the two best fitted critical points are given in Table I.

TABLE I: Best fitted critical points

CP	$(\Omega_\Lambda, \Omega_m, \Omega_b)$	<i>stability</i>
<i>CP1</i>	$(0, 0, 3.5)$	<i>Stable</i>
<i>CP2</i>	$(0, 4.7, 0.9)$	<i>Unstable</i>
<i>CP3</i>	$(0, 0, -0.6)$	<i>SaddlePoint</i>
<i>CP4</i>	$(0, 16.3, 6.7)$	<i>SaddlePoint</i>
<i>CP5</i>	$(0.2, 0.2, -0.5)$	<i>SaddlePoint</i>
<i>CP6</i>	$(0.9 + 2I, 0.2, -0.5 + 0.9I)$	<i>SaddlePoint</i>
<i>CP7</i>	$(0.9 - 2I, 0.2, -0.5 - 0.9I)$	<i>SaddlePoint</i>
<i>CP8</i>	$(3.6, -2.8, -1)$	<i>Unstable</i>
<i>CP9</i>	$(4.8, -3, -0.5)$	<i>Unstable</i>

In Fig. 1, the trajectories leaving the unstable critical point CP1 in the past in the phase plane is shown going towards the stable critical point CP2 in the future. The best fitted model parameter trajectories is also shown by green dashed trajectory.

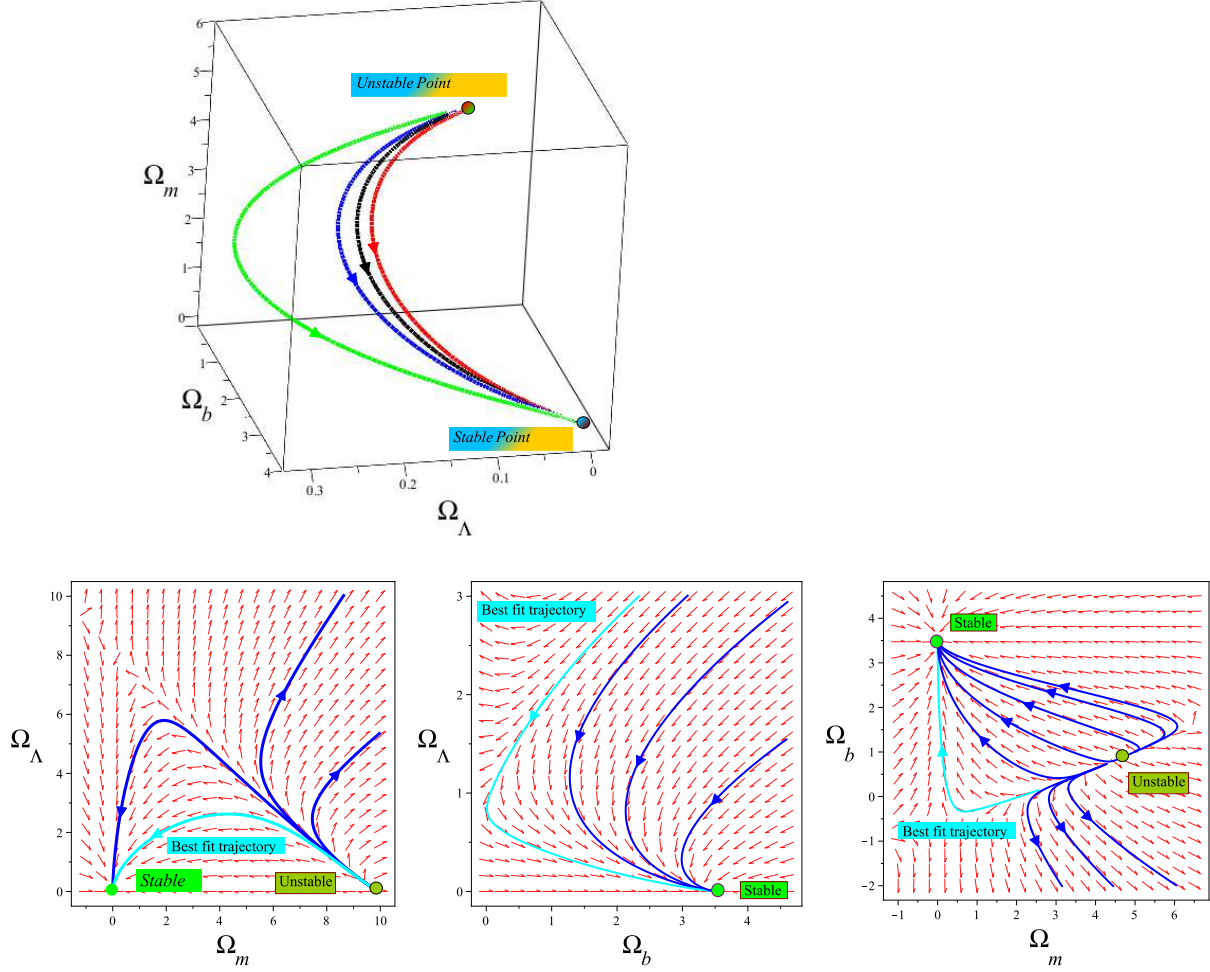


Fig. 1: The 3-dim and 2-dim phase plane corresponding to the critical point

#### 4. COSMOLOGICAL TESTS AND STATEFINDERS

In order to understand the behavior of the universe and its dynamics we need to study the cosmological parameters. We have best fitted our model with the current observational data by the distance modulus test. The cosmological parameters analytically and/or numerically have been investigated by many authors for variety of cosmological models. Simultaneously best fitting the model with the observational data gives us a better understanding of the solutions and the dynamics of these parameters.

We begin with the agegraphic energy density given by equation (28). Taking derivative of



(28) with respect to cosmic time and substituting into conservation equation (13), in terms of new dynamical variables we obtain

$$w_{\Lambda} = -1 - \frac{2\alpha}{3} + \frac{2\sqrt{\Omega_{\Lambda}}}{3n\alpha} - \Omega_b. \quad (28)$$

With the best fitted stability and model parameters, the EoS parameter for agegraphic dark energy can be obtained. One has to know that the contribution of extra dimension into the formalism is via the presence of parameter  $\alpha$  in the field equation for  $\Omega_{\Lambda}$ .

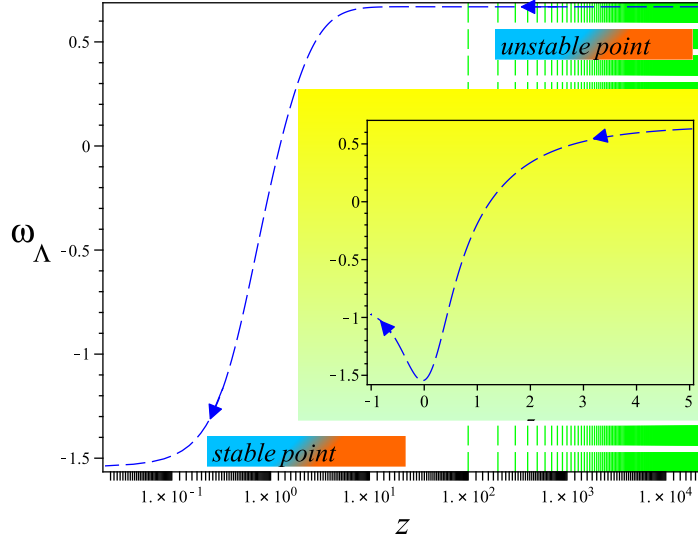


Fig. 2: The best fitted EoS parameter for agegraphic dark energy,  $w_{\Lambda}$ , plotted as functions of redshift

Among cosmological parameters, the effective EoS parameter and statefinders are given by

$$w_{eff} = -1 - \frac{2}{3(1 + \frac{\alpha}{3})} \left[ \left( 1 + \frac{\alpha^2(2 + \omega)}{3} + \alpha + \Omega_m + \Omega_{\Lambda} - 2\alpha \left( 1 - \frac{\alpha\omega}{3} \right) - \Omega_b \left( 1 + \frac{2\alpha}{3} \right) \right] \quad (29)$$

,  $r = \frac{\ddot{H}}{H^3} - 3q - 2$  and  $s = \frac{(r-1)}{3(q-\frac{1}{2})}$  are discussed here, where  $q$  in  $r$  and  $s$  is the deceleration parameter and  $\frac{\ddot{H}}{H^2}$  in  $r$  in terms of new dynamical variables for exponential and power law cases can be obtained by taking derivative of  $\dot{H}$ .

In Table II, the best fitted values of the effective EoS and statefinder parameters of the two critical points are given.

TABLE II: Best fitted values of effective EoS and statefinder parameters

parameter	$w_{eff}$	$q$	$r$	$s$	$\omega_\Lambda$
Current Value	-.85	-.8	-3.2	1.065	-1.52
CP1	-.74	-.61	-2.4	1.065	-.97
CP2	0.46	1.75	3.7	1.415	.67

Fig. 3 shows the dynamics of the best fitted effective EoS parameter against redshift  $z$ . From graph or Table II, the best fitted current value of EoS  $w_{eff}$  is  $-.85$ . The graph also shows that universe undergoes phantom crossing twice in future. From a stability point of view it shows that the universe starts from an unstable state in the higher redshift in the far past and tends to a stable big rip state when the the scale factor of the universe becomes infinite at a finite time in the future.

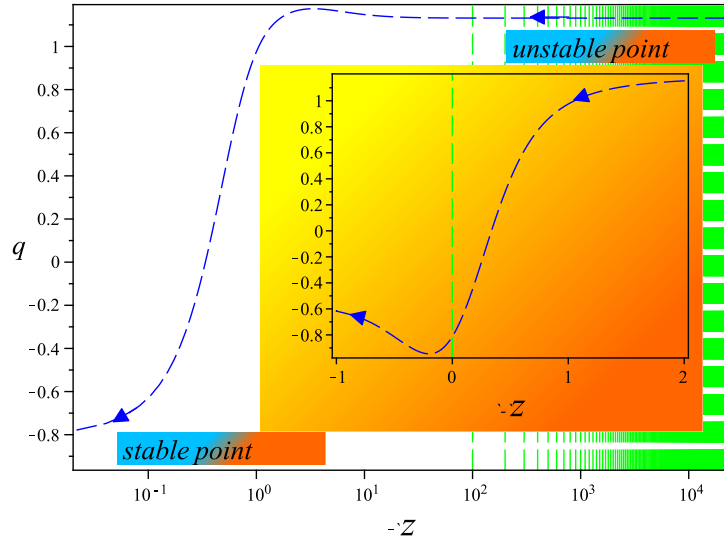


Fig. 3: The best fitted effective EoS parameter  $w_{eff}$ , plotted as functions of redshift, in the solid line) presence and dotted line) absence of the extra dimension.

Fig 4 shows the best-fitted trajectories of the statefinder diagrams  $\{s, q\}$  and  $\{r, s\}$ . From  $\{s, q\}$  graph it can be seen that the best-fitted trajectory is currently between SCDM and SS state with  $\{s, q\} = 1.065, -0.8$ . Also from  $\{r, s\}$  graph we see that the universe passed the LCDM.

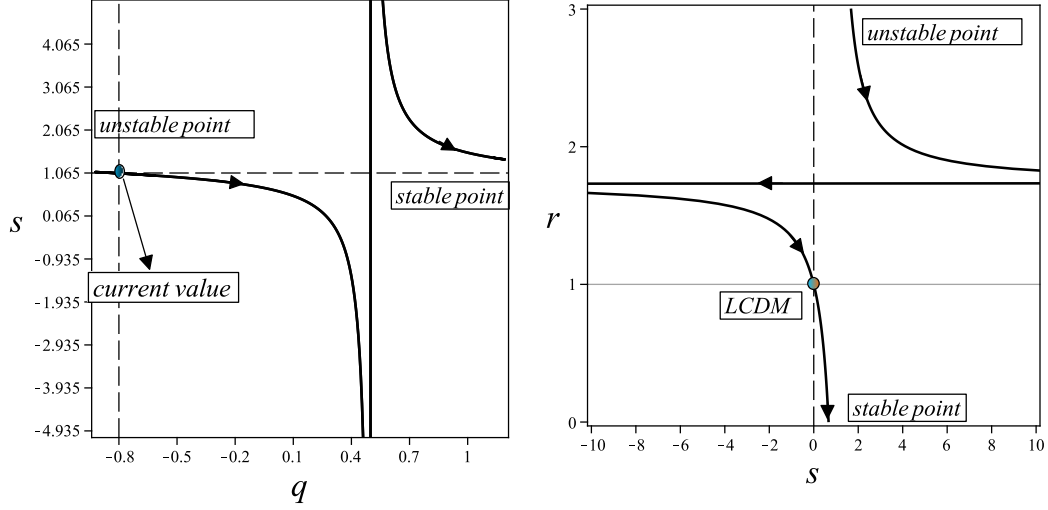


Fig. 4: The best fitted statefinders parameters  $\{s - q\}$  and  $\{r - s\}$

In Fig 5 we depict the corresponding dynamical behavior of the statefinder  $\{r, s\}$  against  $N = \ln(a)$ . From Figs. 4 and 5 we observe that the universe starts its journey from unstable state in the past, passed the current state and eventually reaches a stable state in the future. Note that in Fig 4, the extreme points of the statefinder trajectories corresponds to the state that the universe crosses phantom divide line in future.

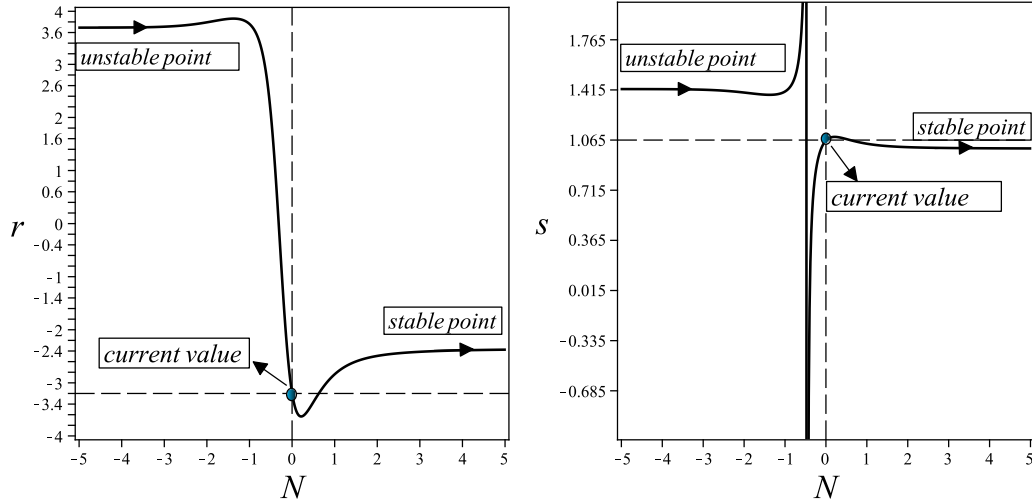


Fig. 5: The best fitted statefinder parameters  $r$  and  $s$  plotted as function of  $N = \ln(a)$ . fig 6 shows the dynamical behavior of the  $\Omega_\Lambda, \Omega_m, \Omega_b$  and  $\Omega_k$  for best fitted both parameters and initial conditions

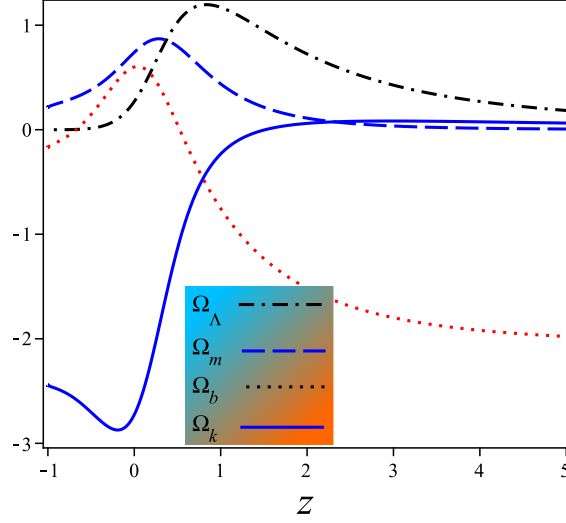


Fig. 6: dynamical behavior of  $\Omega_\Lambda, \Omega_m, \Omega_b$  and  $\Omega_k$  plotted as functions of redshift

Fig.8 shows the dynamic of the  $(\omega_{eff})$  against redshift  $z$  in the presence and absence of the extra dimension. As can be seen from Fig. 7, there is a slight difference between the trajectories of these two cases, we can see from fig 7 that the current value of EoS parameter in absence of extra dimension is  $(\omega_{eff}) \simeq -0.65$  while in presence of extra dimension  $(\omega_{eff}^{current}) \simeq -0.85$  which is more consistent with observational data

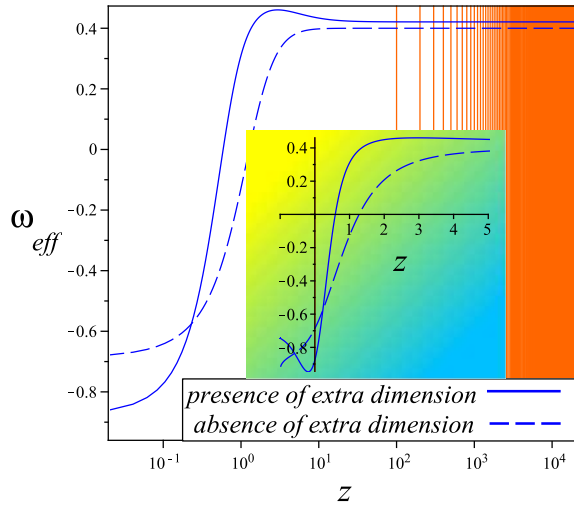
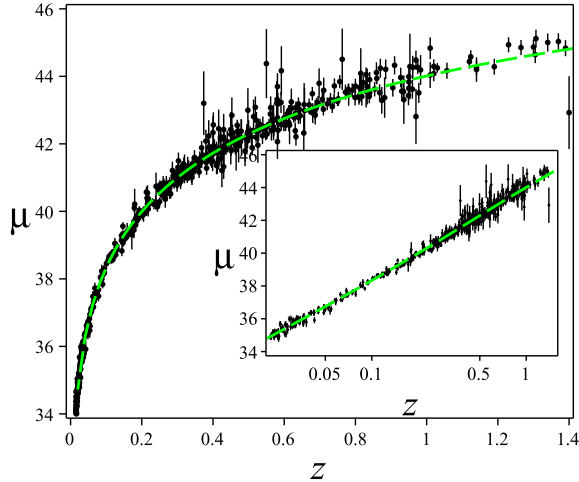


Fig.8: dynamical behavior of effective EoS parameter  $(\omega_{eff})$  plotted as functions of redshift in the solid line): presence of extra dimension and dash line): absence of extra dimension

Fig.9 shows The distance modulus  $\mu(z)$  plotted as function of redshift for the best fitted parameters



The distance modulus  $\mu(z)$  plotted as function of redshift for the best fitted parameters

## 5. SUMMARY AND DISCUSSION

We have constructed an autonomous phase-plane in 5D Brans-Dicke cosmology in the presence of agegraphic dark energy and cold dark matter. We have improved the phase-plane analysis by constraining the stability and model parameters by SNe Ia observational data for distance modulus using  $\chi^2$  method. In a dark energy dominated universe and with the phase space and stability analysis, we find two critical points, and the best fitted trajectory from unstable critical point to the attractor. A quantitative analysis of the dynamical variables and physical parameters in the theory is presented. While both effective and ADE EoS parameters depend on the BD scalar field, model parameter  $n$  and the extra dimension  $b$ , the best fitted trajectories are shown in Fig. 7 in the presence and absence of extra dimension. we see that in presence of extra dimension the dynamical evolution of  $\omega_{eff}$  is more consistent with observational data. The results show that the universe has passed through SCDM and LSDM phase and approaching the SS phase in future.

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